

# 

### Semester One Examination, 2018

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**METHODS**

**UNIT 3**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working  time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question  
have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen (****13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

of the fish in a large inland lake are known to be trout. Ten fish are caught at random from the lake every day.

(a) Describe, with parameters, a suitable probability distribution to model the number of trout in a day's catch. (2 marks)

|  |
| --- |
| **Solution** |
| Binomial, with and |
| **Specific behaviours** |
| ✓ binomial  ✓ parameters |

(b) Determine the probability that there are fewer trout than fish of other species in a day's catch. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes or  ✓ probability, to at least 3dp |

(c) Calculate the probability that over two consecutive days, a total of exactly trout are caught. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ defines new distribution  ✓ probability |

Question 10 (8 marks)

The population of a city can be modelled by , where is the number of people living in the city, in millions, years after the start of the year .

At the start of years and there were and people respectively living in the city.

(a) Determine the value of the constant . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equation  ✓ value of to at least 3sf |

(b) Determine the value of the constant . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equation  ✓ value of (in millions) |

(c) Use the model to determine during which year the population of the city will first exceed . (2 marks)

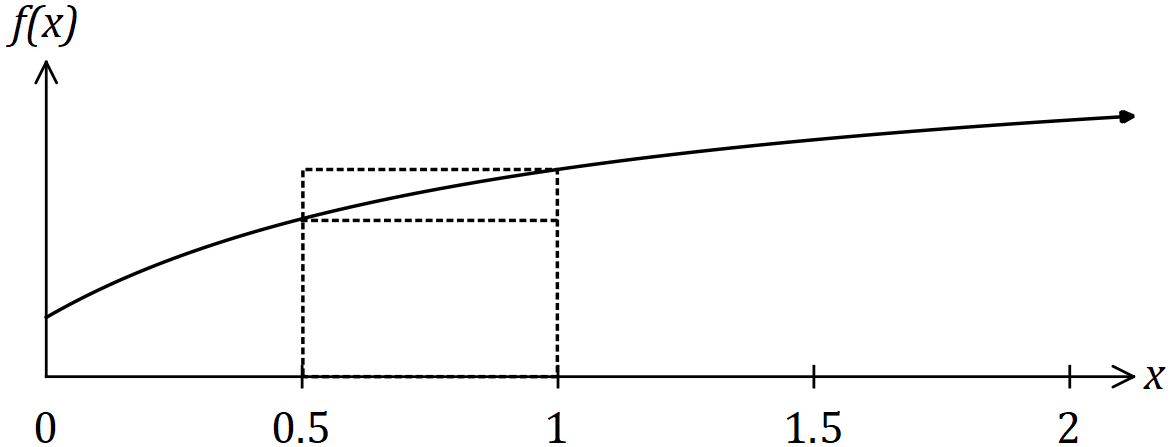
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of  ✓ correct year |

(d) Determine the rate of change of the city's population at the start of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ substitutes into rate of change  ✓ correct rate with units |

Question 11 (6 marks)

The graph of is shown below.



Two rectangles are also shown on the graph, with dotted lines, and they both have corners just touching the curve. The smaller is called the inscribed rectangle and the larger is called the circumscribed rectangle.

|  |
| --- |
| **Solution (a)** |
| See table |
| **Specific behaviours** |
| ✓ missing values |

(a) Complete the missing values in the table below. (1 mark)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

(b) Complete the table of areas below and use the values to determine a lower and upper bound for . (4 marks)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| interval | to | to | to | to |
| Area of inscribed rectangle |  |  |  |  |
| Area of circumscribed rectangle |  |  |  |  |

|  |
| --- |
| **Solution** |
| Lower bound:  Upper bound: |
| **Specific behaviours** |
| ✓ inscribed areas  ✓ circumscribed areas  ✓ states lower bound  ✓ states upper bound |

(c) Explain how the bounds you found in (b) would change if a larger number of smaller intervals were used. (1 mark)

|  |
| --- |
| **Solution** |
| The lower bound would increase and the upper bound decrease. |
| **Specific behaviours** |
| ✓ describes changes to both bounds |

Question 12 (8 marks)

A fairground shooting range charges customers to take shots at a target. A prize of is awarded if a customer hits the target three times and a prize of is awarded if a customer hits the target more than three times. Otherwise no prize money is paid.

Assume that successive shots made by a customer are independent and hit the target with the probability .

(a) Calculate the probability that the next customer to buy shots wins

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ defines distribution  ✓ calculates probability |

(i) a prize of . (2 marks)

(ii) a prize of . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates probability |

(b) Calculate the expected profit made by the shooting range from the next customers who pay for shots at the target. (3 marks)

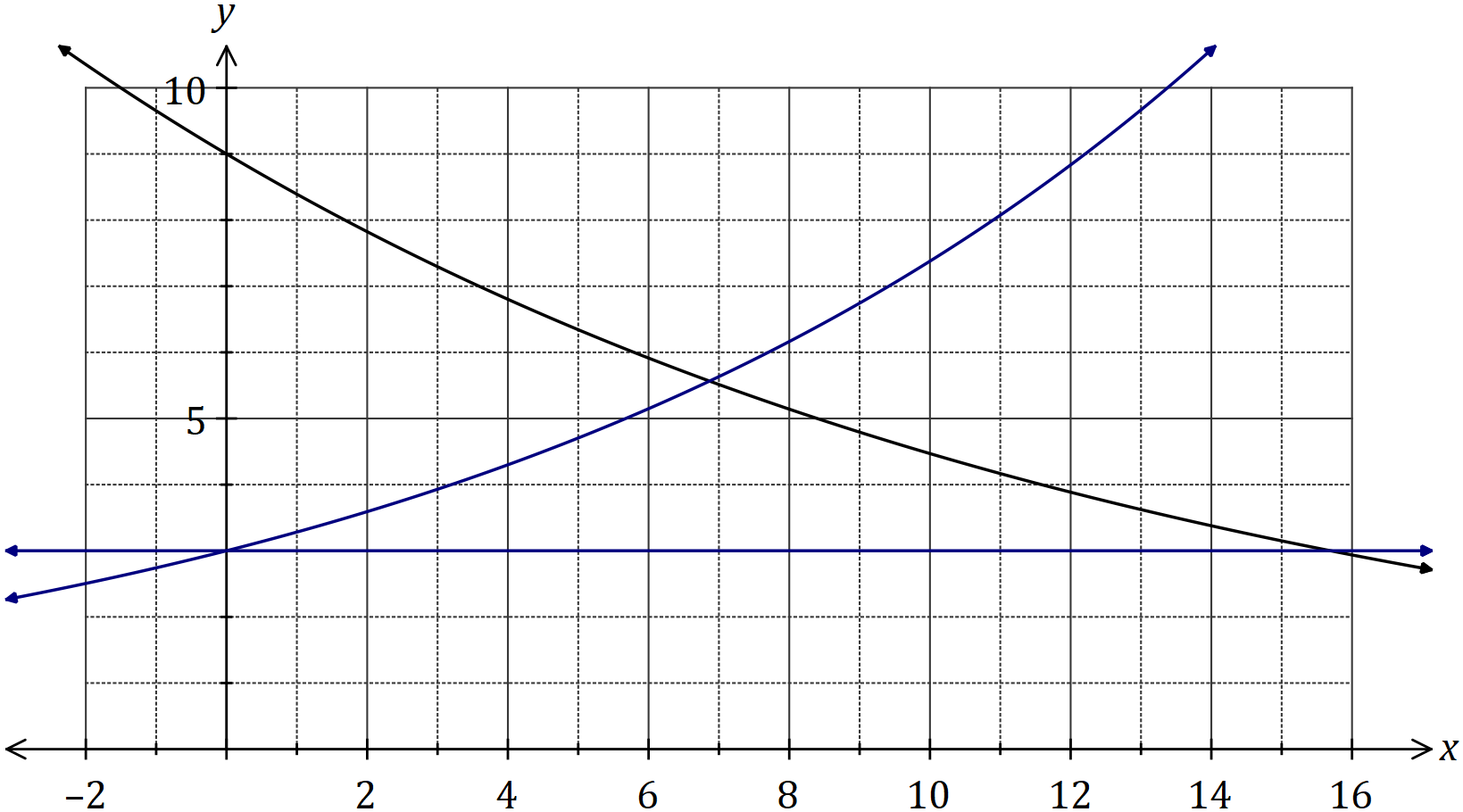
|  |
| --- |
| **Solution** |
| Let be the profit per customer |
| **Specific behaviours** |
| ✓ indicates probability distribution  ✓ calculates expected value for one customer  ✓ calculates expected value |

(c) Determine the probability that more than out of the next customers will not win a prize. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ defines distribution  ✓ calculates probability |

Question 13 (8 marks)

Three functions are defined by and .



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓  ✓ close to and  ✓ smooth curve |

(a) One of the functions is shown on the graph above. Add the graphs of the other two functions. (3 marks)

(b) Working to three decimal places throughout, determine the area of the region enclosed by all three functions. (5 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes first integral  ✓ evaluates first integral  ✓ writes second integral  ✓ evaluates second integral  ✓ total area  *(Rounding instruction supplied for guidance only)* |

Question 14 (7 marks)

A fuel storage tank, initially containing L, is being filled at a rate given by

where is the volume of fuel in the tank in litres and is the time in minutes since filling began. The tank will be completely full after minutes.

(a) Calculate the volume of fuel in the tank after half-an-hour. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates use of integral of rate of change  ✓ calculates increase  ✓ states volume |

(b) Determine the time taken for the tank to fill to three-quarters of its maximum capacity.

(4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates  ✓ indicates  ✓ indicates equation  ✓ solves for time |

Question 15 (8 marks)

The discrete random variable has a mean of and the following probability distribution.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

(a) Determine the values of the constants and . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equation using sum of probabilities  ✓ equation using mean  ✓ values of and |

(b) Determine . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ denominator  ✓ numerator and expresses as decimal or fraction |

(c) Determine

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct variance |

(ii) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct mean |

(iii) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct variance |

Question 16 (9 marks)

A particle starts from rest at and travels in a straight line.

Its velocity ms-1, at time s, is given by for and for .

(a) Determine the initial acceleration of the particle. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ differentiates velocity  ✓ acceleration |

(b) Calculate the change in displacement of the particle during the first three seconds.

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ integrates velocity  ✓ change in displacement |

(c) Determine, in terms of , an expression for the displacement, m, of the particle from for . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ integrates velocity  ✓ evaluates |

(d) Determine the distance of the particle from when its acceleration is ms-2.

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ acceleration for  ✓ solves for time  ✓ calculates distance |

Question 17 (7 marks)

A random sample of components are selected at random from a factory production line. The proportion of components that are defective is and the probability that a component is defective is independent of the condition of any other component.

The random variable is the number of faulty components in the sample. The mean and standard deviation of are and respectively.

(a) Determine the values of and . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates binomial distribution  ✓ equation using mean  ✓ equation using standard deviation  ✓ solves correctly for and |

(b) After changes are made to the manufacturing process, the proportion of defective components is now . Determine the smallest sample size required to ensure that the probability that the sample contains at least one defective component is at least .

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates required binomial probability  ✓ uses to create inequality  ✓ solves and rounds to obtain |

Question 18 (11 marks)

The air pressure, in kPa, experienced by a weather balloon varies with its height above sea level km and is given by

(a) Determine when the height of the balloon is km. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses derivative  ✓ correct rate of change |

(b) What is the meaning of your answer to (a). (1 mark)

|  |
| --- |
| **Solution** |
| The rate of change of pressure with respect to height when the height is km. |
| **Specific behaviours** |
| ✓ meaning *(must include wrt and refer to height)* |

The height of the balloon above sea level varies with time minutes and is given by

(c) Determine the air pressure experienced by the balloon when . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines height  ✓ determines pressure |

(d) Determine when the height of the balloon is km. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines time  ✓ indicates derivative  ✓ determines rate of change |

(e) Determine when the height of the balloon is km. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ rate of change of wrt  ✓ indicates use of chain rule  ✓ correct rate of change |

Question 19 (7 marks)

The hourly cost of fuel to run a train is proportional to the square of its speed and is per hour when the train moves at a speed of kmh-1. Other costs amount to per hour, regardless of speed.

(a) Show that when the train moves at a steady speed of kmh-1, where , the total cost per kilometre, , is given by (3 marks)

|  |
| --- |
| **Solution** |
| Fuel cost, , is  Total cost per hour, , is  Cost per km, , is |
| **Specific behaviours** |
| ✓ expression for hourly cost of fuel  ✓ expression for total cost per hour  ✓ indicates derivation of cost per km |

(b) Use calculus to determine the minimum cost for the train to travel km, assuming that the train travels at a constant speed for the entire journey. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ obtains first derivative  ✓ indicates critical point  ✓ indicates optimum cost per km  ✓ correct minimum cost |

Question 20 (6 marks)

The discrete random variable is defined by

(a) Show that . (3 marks)

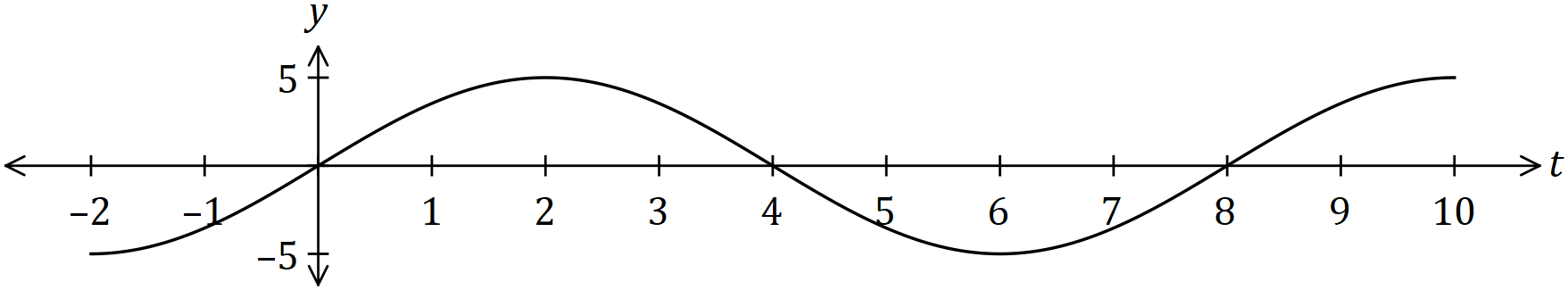
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates and  ✓ sums probabilities to 1  ✓ factors out and rearranges |

(b) Determine, in simplest form, the exact mean and standard deviation of . (3 marks)

|  |
| --- |
| **Solution** |
| NB Bernoulli distribution. |
| **Specific behaviours** |
| ✓ simplified  ✓ correct expression for variance  ✓ simplified expression for standard deviation |

Question 21 (7 marks)

The graph of is shown below, where .



(a) Determine the exact area between the horizontal axis and the curve for .

(2 marks)

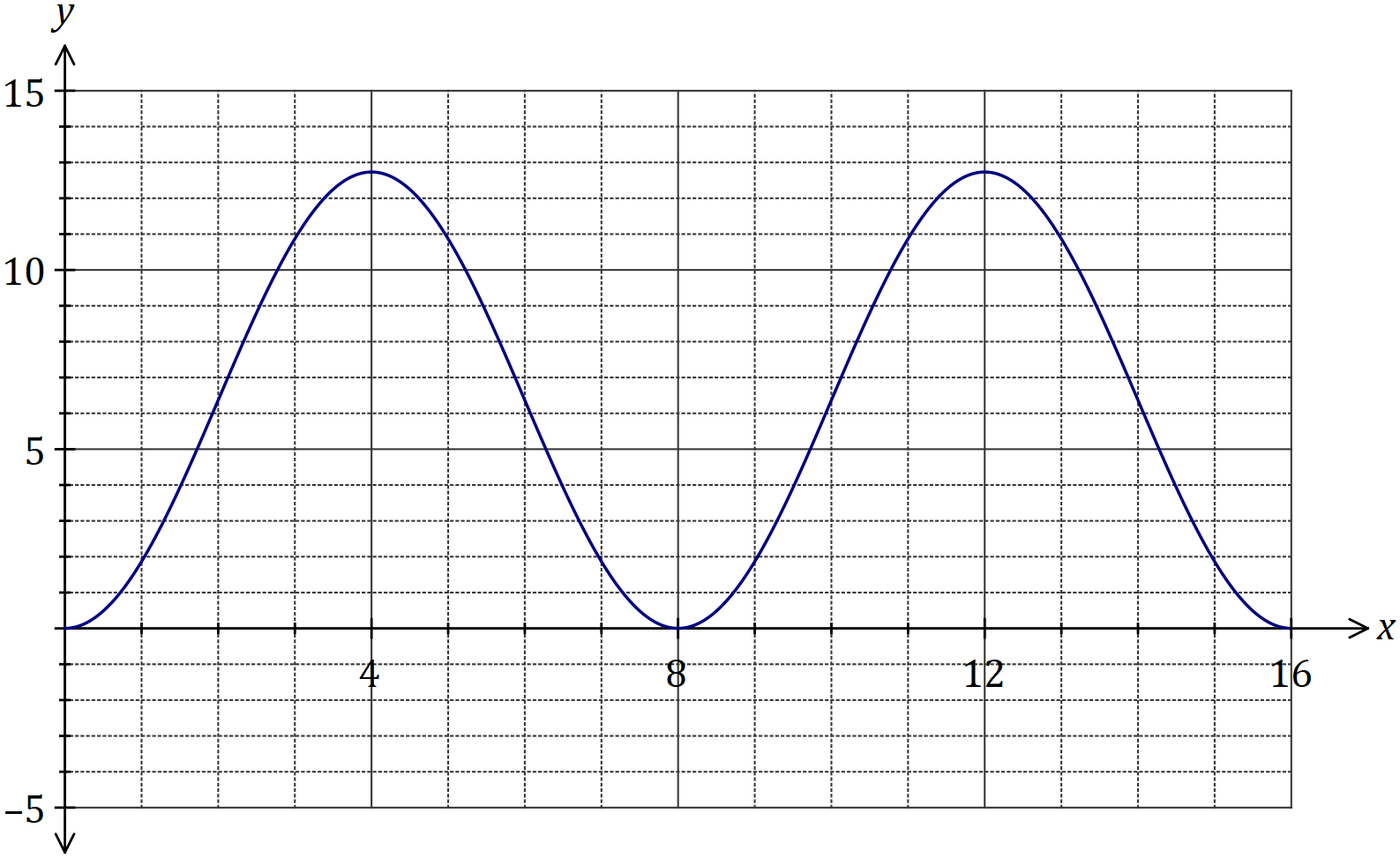
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes integral  ✓ evaluates |

Another function, , is defined as over the domain .

(b) Determine the value(s) of for which has a maximum and state the value of at this location. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ values of  ✓ value of |

(c) Sketch the graph of on the axes below. (3 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ maxima  ✓ minima  ✓ smooth continuous curve |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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